Sequences and Series

Question1

If 4^{th} , 10^{th} and 16^{th} terms of a G.P. are x, y and z respectively, then

KCET 2025

Options:

A. $z=\sqrt{xy}$

B. $y=\sqrt{xz}$

C. $x=\sqrt{yz}$

D. $y=\frac{x+z}{2}$

Answer: B

Solution:

In a G.P. the nth term is ar^{n-1}. Thus

4th term: $x=ar^3$

10th term: $y=ar^9$

16th term: $z=ar^{15}$

Now

$xz=(ar^3)(ar^{15})=a^2r^{18}=(ar^9)^2=y^2$

so

$y^2=xz \implies y=\sqrt{xz}.$

Answer: Option B.

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Question2

If S_n stands for sum to n-terms of a GP with a as the first term and r as the common ratio, then $S_n : S_{2n}$ is

KCET 2024

Options:

A. $r^n + 1$

B. $\frac{1}{r^n+1}$

C. $r^n - 1$

D. $\frac{1}{r^n-1}$

Answer: B

Solution:

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S_{2n} = \frac{a\left(1-r^{2n}\right)}{1-r}$$

$$\frac{S_n}{S_{2n}} = \frac{a\left(1-r^n\right)}{1-r} \times \frac{1-r}{a\left(1-r^{2n}\right)} = \frac{1-r^n}{\left(1+r^n\right)\left(1-r^n\right)}$$

Hence, $S_n : S_{2n} = \frac{1}{1+r^n}$

Question3

If $p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$ are in AP, then p, q, r

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Options:

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A. are in GP

B. are not in GP

C. are in AP

D. are not in AP

Answer: B

Solution:

$$p\left(\frac{1}{q}+\frac{1}{r}\right), q\left(\frac{1}{r}+\frac{1}{p}\right), r\left(\frac{1}{p}+\frac{1}{q}\right)$$
$$=p\left(\frac{q+r}{qr}\right), q\left(\frac{p+r}{pr}\right), r\left(\frac{p+q}{pq}\right)$$
$$=\frac{p(q+r)}{qr}+1, \frac{q(p+r)}{pr}+1, r\left(\frac{p+q}{pq}\right)+1 \quad \text{On adding 1 each term)}$$

$=\frac{pq+pr+qr}{qr}, \frac{qp+qr+pr}{pr}, \frac{rp+rq+pq}{pq}$

On dividing $pq+pr+qr$ and term, we get

$$=\frac{1}{qr}, \frac{1}{pr}, \frac{1}{pq} \text{ in AP}$$
$$=\frac{pqr}{qr}, \frac{pqr}{pr}, \frac{pqr}{pq} \text{ in APs} \quad \text{(on multiply pqr both order)}$$

$\Rightarrow p, q, r$ also in AP.

Question4

nth term of the series $1+\frac{3}{7}+\frac{5}{7^2}+\frac{1}{7^2}+\ldots$ is

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Options:

A. $\frac{2n+1}{7^n}$

B. $\frac{2n-1}{7^n}$

C. $\frac{2n+1}{7^{n-1}}$

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D. $\frac{2n-1}{7^{n}-1}$

Answer: D

Solution:

We have,

$$1+\frac{3}{7}+\frac{5}{72}+\frac{7}{73}+\ldots$$
$$\frac{1}{70}+\frac{3}{7}+\frac{5}{72}+\frac{7}{73}+\ldots$$

Numerator are in AP

So, T_n of AP $= 1+(n-1)^2 = 2n-1$

Denominator are in GP

So, T_n of GP $= 7^{n-1}$

T_n of given series $= \frac{2n-1}{7^{n-1}}$

Question5

If $A=\{1,2,3,\ldots,10\}$, then number of subsets of A containing only odd numbers is

KCET 2022

Options:

A. 31

B. 27

C. 32

D. 30

Answer: A

Solution:

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Given, $A = \{1, 2, 3, \ldots, 10\}$

Set A containing only odd number $= \{1, 3, 5, 7, 9\}$

Number of subsets $= 2^5$ [it contains empty set also]

The required number of subsets of A containing only odd number $= 2^5 - 1$

$= 32 - 1 = 31$

Question6

If $a_1, a_2, a_3, \ldots, a_{10}$ is a geometric progression and $\frac{a_3}{a_1} = 25$, then $\frac{a_9}{a_5}$ equals

KCET 2022

Options:

A. $3\left(5^2\right)$

B. 5^4

C. 5^3

D. $2\left(5^2\right)$

Answer: B

Solution:

Given, GP : $a_1, a_2, a_3, \ldots, a_{10}$

and $\frac{a_3}{a_1} = 25$

$\Rightarrow \frac{ar^2}{a} = 25 \Rightarrow r^2 = 25$

Now, $\frac{a_9}{a_5} = \frac{ar^8}{ar^4} = r^4 = \left(r^2\right)^2 = (25)^2 = 5^4$

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Question7

If the set x contains 7 elements and set y contains 8 elements, then the number of bijections from x to y is

KCET 2022

Options:

A. 0

B. $8P_7$

C. 7!

D. 8!

Answer: A

Solution:

Number of bijections from x to y will be zero because only sets with the same cardinality have bijections between them and it is given that number of elements in x and y are not same.

Question8

If the middle term of the AP is 300, then the sum of its first 51 terms is

KCET 2021

Options:

A. 15300

B. 14800

C. 16500

D. 14300

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Answer: A

Solution:

Given, number of terms, $n=51$

$\because n$ is odd.

$\therefore$ Middle term will be $\left(\frac{n+1}{2}\right)$th term.

$$= \left(\frac{51+1}{2}\right) \text{ th term } = 26\text{th term}$$

$$\therefore \quad T_{26}=300$$

$a+25d=300 \quad [\because T_n=a+(n-1)d]$

$\Rightarrow \quad T_1+25d=300 \quad [\because T_1=a]$

$\Rightarrow \quad T_1=300-25d$

and $T_{51}=a+50d=T_1+50d=300-25d+50d$

$=300+25d$

$$\therefore S_{51}=\frac{51}{2}[300-25d+300+25d]$$

$$=\frac{51}{2}[600]=15300$$

Question9

If the sum of n terms of an AP is given by $S_n=n^2+n$, then the common difference of the AP is

KCET 2020

Options:

A. 4

B. 1

C. 2

D. 6

Answer: C

Solution:

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Given sum of n terms of an AP.

$$\begin{aligned} S_n &= n^2 + n \\ a_1 &= S_1 = 1 + 1 = 2 \\ a_1 + a_2 &= S_2 = 2^2 + 2 = 6 \\ a_2 &= S_2 - S_1 = 6 - 2 = 4 \\ d &= a_2 - a_1 = 4 - 2 = 2 \end{aligned}$$

Question10

The third term of a GP is 9. The product of its first five terms is

KCET 2019

Options:

A. 3^{10}

B. 9^5

C. 3^{12}

D. 3^9

Answer: B

Solution:

Let first term and common ratio of a GP are a and r respectively

$\therefore \quad ar^2 = 9 \quad$ ..... (i)

then product of first five terms is GP

$$\begin{aligned} &= (a)(ar)\left(ar^2\right)\left(ar^3\right)\left(ar^4\right) \\ &= a^5 r^{10} = \left(ar^2\right)^5 = (9)^5 (\text{ by using (i)}) \end{aligned}$$

Question11

If a, b, c are three consecutive terms of an AP and x, y, z are three consecutive terms of a GP, then the value of

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$x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$ **is**

KCET 2018

Options:

A.

0

B.

-1

C.

xyz

D.

1

Answer: D

Solution:

Given that a, b, and c are three consecutive terms of an arithmetic progression (AP), and x, y, and z are three consecutive terms of a geometric progression (GP), we want to find the value of the expression $x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$.

Analysis:

For an AP, the difference between consecutive terms is constant. Thus, if a, b, and c are in AP, we have:

$b - a = c - b = d$

Thus, $c - a = 2d$.

For a GP, the ratio between consecutive terms is constant. Hence, if x, y, and z are in GP, then:

$\frac{y}{x} = \frac{z}{y} = r \quad \Rightarrow \quad y^2 = xz$

Now, let's simplify the expression $x^{b-c} \cdot y^{c-a} \cdot z^{a-b}$:

$$\begin{aligned} x^{b-c} \cdot y^{c-a} \cdot z^{a-b} &= x^{-(c-b)} \cdot y^{2d} \cdot z^{-(b-a)} \\ &= x^{-d} \cdot y^{2d} \cdot z^{-d} \\ &= (x \cdot z)^{-d} \cdot y^{2d} \end{aligned}$$

Since $y^2 = xz$, it follows that:

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$y^{-2d} \cdot y^{2d} = 1$

Therefore, the value of the expression is:

$x^{b-c} \cdot y^{c-a} \cdot z^{a-b} = 1$

Question12

$3 + 5 + 7 + \ldots$ to n terms is

KCET 2017

Options:

A. $n(n+2)$

B. $(n+1)^2$

C. n^2

D. $n(n-2)$

Answer: A

Solution:

The sequence given is:

$S_n = 3 + 5 + 7 + \ldots$

This sequence is an arithmetic progression (AP) where:

The first term $a = 3$

The common difference $d = 5 - 3 = 2$

To find the sum of the first n terms (S_n) of this AP, we use the formula for the sum of an arithmetic sequence:

$S_n = \frac{n}{2}[2a + (n-1)d]$

Substitute the known values into the formula:

$S_n = \frac{n}{2}[2 \times 3 + (n-1) \times 2]$

Simplify the expression:

$= \frac{n}{2}[6 + 2n - 2]$

$= \frac{n}{2}[2n + 4]$

$= n(n+2)$

Thus, the sum of the first n terms of the sequence is $n(n+2)$.

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